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Analysis of reliability and availability of serial processes of plastic-pipe manufacturing plant

A case study

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Abstract

Purpose – The purpose of this paper is to compute reliability, availability, and mean time before failure of the process of a plastic-pipe manufacturing plant consisting of a (*K, N*) system for various choices of failure and repair rates of sub-systems. This plant consists of eight sub-systems.

Design/methodology/approach – In this paper the Chapman-Kolmogorov differential equations are formed using mnemonic rule from the transition diagram of the plastic-pipe manufacturing plant. The governing differential equations are solved using matrix method in order to find the reliability of the system with the help of MATLAB software. The same system of differential equations is solved numerically using Runge-Kutta fourth order method to validate the results obtain by MATLAB.

Findings – The findings in the paper are an analysis of reliability, availability and mean time before failure of plastic-pipe manufacturing plant has been carried out.

Practical implications – This paper proposes matrix calculus method using MATLAB software to find out the reliability of the plastic-pipe manufacturing plant. This approach can be implemented to find reliability of other manufacturing plants as well.

Originality/value – The findings suggest that the management of the plastic-pipe manufacturing plant 's sensitive sub-system is important to improve its performance.

Keywords Modelling, Differential equations, Reliability management, Mean time between failures, Pipes

Paper type Research paper

1. Introduction

System reliability is an important issue in evaluating the performance of an engineering system. With the increase in the complexity of a system, the reliability will decrease unless some precautionary measures are taken into account. High reliability can be achieved either by providing sufficient redundant parts or by increasing capacity of the system. To save time and expenses, one may allow several imperfect repairs before the system is replaced with a new one, or is perfectly repaired. In order to

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discuss the reliability of a system, we assume that it is being continuously monitored. As soon as the system fails, a repair work starts on it and when the repair is completed, the system is reinstated for operation. The purpose is to bring the failed system back to a functioning state as soon as possible. Although replacement by a new unit is the quickest solution, it is not always desirable as it is also the costliest solution. Therefore, reuse of the old unit after repair is almost mandatory. We thus need to study the reliability and availability of the system under these considerations. These issues have been discussed by several authors, in their research papers, using different techniques. After getting the governing differential equations of any system by applying the mnemonic rule on the transition diagram of the system, they applied different mathematical approaches to solve these equations for finding reliability. Singh and Dayal (1991) discussed one-out of *N:G* system with common cause failure and critical human errors where; Shao and Lamberson (1991) studied modeling of a shared load *K*-out of *N:G* system. The reliability of a system in fluctuating environment, was discussed by Dayal and Singh (1992). Yang and Dhillon (1995) studied availability analysis of a repairable standby human-machine system; Subramaniam and Ananthraman (1995) discussed the reliability of complex redundant system; Kansal *et al.* (1995) studied reliability of water distribution under uncertainty. Galikowsky *et al.* (1996) discussed optimal redundancies for reliability and availability of series system. Sridharan and Mohanavadivu (1997) studied the reliability for two non-identical unit parallel systems with common cause failures and human errors; Tin (1997) analysed reliability and availability of two units warm standby microcomputer system with self-reset function and repair facility. Biswas and Sarkar (2000) studied availability of a system maintained through several imperfect repairs before a replacement or a perfect repair. Availability of a periodically inspected system supported by a spare unit, under perfect repair, and, perfect upgrade was discussed by Sarkar and Sarkar (2001). Dai *et al.* (2003) studied the service reliability and availability for distribution system; Jain (2003) discussed *N* policy for redundant repairable system with additional repairman. Madu (2005) analysed strategic value of reliability and maintainability management. Most of these authors used Laplace transform method or Lagrange's method to find out reliability of the system. It has been observed that taking Laplace inverse of higher order terms or solving higher order integrals is a complex phenomenon and as such these methods are not so preferable. Singh (1975) discussed the reliability of multiple parallel channels with finite sources, and used matrix method to solve the governing differential equations. Finding the eigenvalues and eigen-vectors of higher order matrix that is produced after mathematical formulations becomes a bottleneck in the implementation of this method.

In the present research paper we have used matrix method to solve the governing differential equations formulated for the serial process of plastic-pipe manufacturing plant with the help of MATLAB and the reliability of the system has been analysed for different failure and repair rates of its sub-systems. The results thus obtained are also verified with the help of Runge-Kutta fourth order method earlier applied by Gupta *et al.* (2005) for numerical analysis of reliability and availability of the serial processes in butter-oil processing plant.

Plastic-pipes are produced by an extrusion process. An engineering explanation of plastic-pipe extrusion is that in it plastic granules are heated and melted, then mixed and formed into pipe. This plant consists of mixer, extruder, extrusion die, calibration

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bush sleeve, spray cooling bath, haul-off, cutting saw, and trifling chute. Figure 1 gives the flow chart of the process of plastic-pipe manufacturing plant.

This paper has been organized in five sections. The present section is introductory in nature. In section 2, a brief description about system and its notations are presented. Certain assumptions, on which the present analysis is based, are also given in this section. The main differential equations governing reliability and long run availability are discussed in section 3. In section 4, effect of failure and repair rates of the sub-system on reliability and long run availability of the plant have been studied and section 5 gives the analysis of these results for the plastic-pipe manufacturing plant.

2. System, notations and assumptions

In the present paper, we have considered a plastic-pipe manufacturing plant, which consists of eight sub-systems. The sub-systems, calibration bush sleeve, extrusion die, and trifling chute never fail. The remaining sub-systems contribute in the mathematical formulations.

(1) *Sub-system A (Mixer):* It mixes raw material such as calcium carbonate, wax, and other chemicals in appropriate proportion for manufacturing pipes. It consists of a heater by which the raw material is heated up to 140°C. The heated material is then cooled up to 100°C and transported to the extruder by conveyors. This sub-system is subject to major failure.

(2) *Sub-system B (Extruder):* Raw material from mixer is heated in this sub-system. It consists of nine heaters to heat the raw material at different temperatures. The quality of the product depends upon heating process. If one or two heaters fail, then it does not affect the working of the sub-system. However, if more than two heaters fail, the sub-system fails. This sub-system is also subject to major failure.

(3) *Sub-system C (Extrusion Die):* It is used to make pipes of different sizes. It is supported by a standby sub-system. As such, it can be considered that it never fails.

(4) *Sub-system D (Calibration Bush Sleeve):* It is used to reduce the temperature of hot pipe. It also has a standby sub-system and thus this sub-system never fails.

(5) *Sub-system E (Spray Cooling Bath):* In this sub-system, we have $N(=10)$ taps in parallel for cooling the pipe. It fails only when $K(=2)$ taps out of N fail.

(6) *Sub-system F (Haul-Off):* It is a three-unit system having the units in series subject to major failure only. This sub-system is used to pull the pipes from cooling bath and to push them to cutting saw.

(7) *Sub-system G (Cutting Saw):* This is a two-unit system having the units in series. One unit is a blade to cut the pipe and the second is a motor. If any one of these two units fails, the system fails. This sub-system is used to cut the pipe in required length and is subject to major failure.

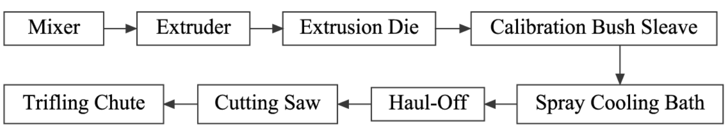

Figure 1.
Flow chart of plastic-pipe manufacturing plant

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(8) *Sub-system H (Trifling Chute)*: It is used to measure the length of pipe and is also responsible for collecting the pipes. Based upon the feedback from plant engineers, it has been considered that this sub-system can also be considered to never fail.

In addition to the notations used for good states of sub-systems as *A, B, C, D, E, F, G* and *H*, we have also used the following notations:

α_i: Constant failure rates of sub-systems *A, B, F* and *G*, respectively, $i = 1, 2, 3, 4$.

γ: Constant failure rate of sub-system *E*.

β_i: Constant repair rates of sub-systems *A, B, F* and *G*, respectively, $i = 1, 2, 3, 4$.

δ: Constant repair rate of sub-system *E*.

$P_j(t)$: Probability that the system is in j^{th} state at time $t, j = 1, 2, \ldots, 11$.

Small letters *A, B, E, F* and *G* indicate the failed states of the respective sub-systems *A, B, E, F* and *G*.

Assumptions

- Repair and failure rates are independent of each other and their unit is taken as per day.
- The failure and repair times are random and are arbitrarily distributed.
- All system units are identical.
- The occurrence of a common failure or human error results in the failure of the entire system from its operable state. The repair process begins soon after a unit fails.
- There are no simultaneous failures among the sub-systems.
- No further failure can occur when system is in failure state.
- The switchover devices used for standby sub-systems are perfect and hence the sub-systems *C, D* and *H* never fail.
- Sub-system *E* fails only when $K(=2)$ taps out of $N(=10)$, fail.
- The repaired unit or system is as good as new.

Using the above assumptions and notations, transition diagram of complete system is given in Figure 2.

3. Mathematical formulation of the system

To determine the reliability and long run availability of the plastic-pipe manufacturing plant, the mathematical formulation is carried out by using mnemonic rule for eight sub-systems of the plant.

3.1 Transient state

In order to find reliability of this system, we have formed a system of linear differential equations using mnemonic rule. This rule states that the derivative of the probability of every state is equal to the sum of all probability flows which comes from other states

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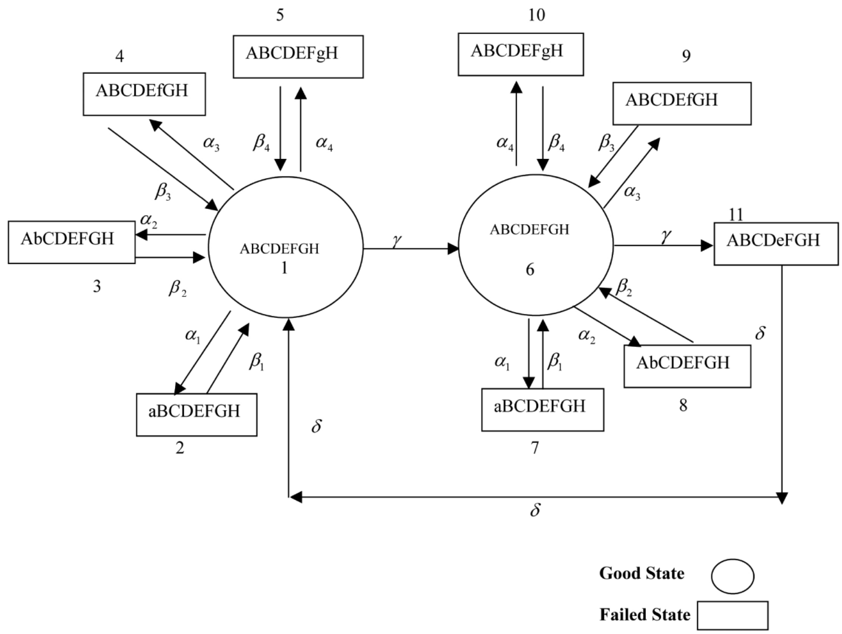

Figure 2.
Transition diagram of the plastic-pipe manufacturing plant

to the given state minus the sum of all probability flows which goes out from the given state to the other states. The differential equations thus derived are known as the Chapman-Kolmogorov differential equations. Probability considerations, using this rule on transition diagram of the system, give the following system of first order differential equations at time $(t + \Delta t)$. The differential equation for state 1 can be written as,

$$P'_1(t) = \beta_1 P_2(t) + \beta_2 P_3(t) + \beta_3 P_4(t) + \beta_4 P_5(t) + \delta$$
$$P_{11}(t) + [-\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \gamma] P_1(t)$$

or

$$P'_1(t) + \lambda P_1(t) = \beta_1 P_2(t) + \beta_2 P_3(t) + \beta_3 P_4(t) + \beta_4 P_5(t) + \delta P_{11}(t) \qquad (1)$$

where,

$$\lambda = \gamma + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

The differential equation for state 6 can be written as

$$P'_6(t) = \gamma P_1(t) + \beta_1 P_7(t) + \beta_2 P_8(t) + \beta_3 P_9(t) + \beta_4$$
$$P_{10}(t) + [-\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \gamma] P_6(t) \qquad (2)$$

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or

$$P'_6(t) + \lambda P_6 = \gamma P_1(t) + \beta_1 P_7(t) + \beta_2 P_8(t) + \beta_3 P_9(t) + \beta_4 P_{10}(t)$$

Similarly, for the other states, we can write the differential equations as,

$$P'_{i+1}(t) + \beta_i P_{i+1}(t) = \alpha_i P_1(t), i = 1, 2, 3, 4 \tag{3}$$

$$P'_{i+6}(t) + \beta_i P_{i+6}(t) = \alpha_i P_6(t), i = 1, 2, 3, 4 \tag{4}$$

$$P'_{11}(t) + \delta P_{11}(t) = \gamma P_6(t) \tag{5}$$

$$\text{The initial conditions are } P_i(0) = 1, \text{ for } i = 1, 6 \text{ and } 0 \text{ otherwise} \tag{6}$$

The above system of equations may be written as,

$$(\theta I - A)P_k(t) = 0 \tag{7}$$

where $\theta \equiv \frac{d}{dt}$, is the differential operator, **(** is an 11-identity matrix and ***0*** is an 11-null matrix, $[P_k(t) = \left[P_1(t), P_2(t), \ldots, P_{11}(t)\right]^T$ and ***A*** is a 11-square matrix defined as,

$$A = \begin{bmatrix} -\lambda & \beta_1 & \beta_2 & \beta_3 & \beta_4 & 0 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -\beta_1 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_2 & 0 & -\beta_2 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_3 & 0 & 0 & -\beta_3 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_4 & 0 & 0 & 0 & -\beta_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma & 0 & 0 & 0 & 0 & -\lambda & \beta_1 & \beta_2 & \beta_3 & \beta_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_1 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_2 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 & 0 & -\beta_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_4 & 0 & 0 & 0 & -\beta_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & -\delta \end{bmatrix}$$

Let ***C*** be the transformation matrix of ***A***, such that

$$C^{-1}AC = D \tag{8}$$

where, ***D*** = **diag** $(d_1, d_2, \ldots, d_{11})$; $d_1, d_2, \ldots, d_{11}$ being the eigenvalues of ***A***.

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Pre-multiplying (7) by C^{-1} and using (8) we get,

$$(\theta I - D)G_k(t) = 0 \tag{9}$$

where, $G_k(t)=C^{-1}\,P_k(t)$

If we can compute $G_k(t)$, then $P_k(t)$ can be determined by the transformation,

$$P_k(t) = CG_k(t) \tag{10}$$

Equation (9) is a linear matrix differential equation in $G_k(t)$, whose solution is given by

$$G_k(t) = \exp(D_t)L \tag{11}$$

where, $L = G_k(0) = C^{-1}\,P_k(0)$ and $P_k(0)$ is a column vector, all of whose elements are 0 except the first element, which is 1 (using the initial conditions). Hence, we get,

$$P_k(t) = C\exp(D_t)C^{-1}P_k(0) \tag{12}$$

Thus, all the probabilities can be obtained, as polynomials in t, which in turn will give reliability of the system for a given value of t. The system of differential equations (1) to (5) with given initial conditions (6) has been solved with this method and the reliability of the system then computed for the time, $t = 0$ to $t = 360$ days for different choices of repair and failure rates of the sub-systems. It is evident that reliability $R(t)$ of the system can be computed as,

$$R(t) = P_1 + P_6 \tag{13}$$

3.2 Steady state

In process industries, management is generally interested in the long run availability of the system. So the steady state probabilities of the system are also needed. Steady state probabilities of the system are obtained by taking,$\frac{d}{dt}\rightarrow 0$ as $t\rightarrow\infty$. In this limiting case, equations (1) to (5) reduce to the following equations. Here, we have used P_j for$P_j(t\rightarrow\infty)$, $j = 1, 2, \ldots, 11$ for denoting steady state probabilities γ.

$$\lambda P_1 = \sum_{j=1}^{4}\beta_j P_{j+1} + \delta P_{11} \tag{14}$$

$$\lambda P_6 = \sum_{j=1}^{4}\beta_j P_{j+6} + \gamma P_1 \tag{15}$$

$$\beta_j P_{j+1} = \alpha_j P_1 \tag{16}$$

$$\beta_j P_{j+6} = \alpha_j P_6 \tag{17}$$

where $j = 1, 2, 3, 4$

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and

$$\delta P_{11} = \gamma P_6 \tag{18}$$

Solving these equations recursively, we obtain:

$$P_{j+1} = \tfrac{\alpha_j}{\beta_j} P_1, j = 1,2,3,4. \ P_{j+6} = \tfrac{\alpha_j}{\beta_j} P_6, j = 1,2,3,4.$$
$$P_{11} = \tfrac{\gamma}{\delta} P_6, \tag{19}$$

Now, using normalizing condition, $\sum_{j=1}^{11} P_j = 1$, we get

$$P_1 = \frac{1}{2}\left[1 + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + \frac{\gamma}{2\delta}\right]^{-1}$$

The long run long run availability can now be obtained as,

$$A(\infty) = P_1 + P_6$$

$$= \left[1 + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + \frac{\gamma}{2\delta}\right] \tag{20}$$

4. Behaviour study

In this section, we calculate the reliability for transient state using matrix method and long run availability for the steady state recursively of the system based on observed values of failure and repair rates of sub-systems.

4.1 Computations and results for transient state

The reliability of the system as defined in equation (13) has been computed for various value-combinations of the repair and failure rates. It may be mentioned here that these combinations are not exhaustive and we have only considered the main sub-systems in the numerical study. The reliability of the system based on different value-combinations of the failure and repair rates is calculated using matrix calculus method with the help of MATLAB and is presented in Tables I, II, III, IV, V, VI. Table VII presents the effect of repair rate of spray cooling bath on reliability of the system, when calculated by Runge-Kutta method. The last row of these tables gives the mean time before failure (*MTBF*) in days for the respective failure rates. *MTBF* has been computed by using Simpson's one-third rule.

Effect of failure rate of mixer (α_1) on the reliability of the system. Taking α_1, 0.0027, 0.0029 and 0.0030 and other parameters as: $\alpha_2 = 0.005$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$, $\gamma = 0.0054$, $\beta_1 = 4.800$, $\beta_2 = 0.033$, $\beta_3 = 0.0999$, $\beta_4 = 0.4999$ and $\delta = 12.000$, the reliability of the system has been calculated. The results of this study are tabulated in Table I. The table shows that the reliability and *MTBF* decreases by approximately 0.01 percent with the increase in failure rate from 0.0025 to 0.0030. Reliability decreases by approximately 0.6 percent with increase in time from 30 to 360 days.

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Table I.
Effect of failure rate of mixer on reliability of the system

$\alpha_1 \rightarrow$ Days $\downarrow$	0.0025	0.0027	0.0029	0.0030
30	0.979490	0.979450	0.979410	0.979390
60	0.975678	0.975639	0.975599	0.975579
90	0.974400	0.974361	0.974321	0.974301
120	0.973927	0.973888	0.973849	0.973829
150	0.973748	0.973708	0.973668	0.973648
180	0.973675	0.973636	0.973596	0.973576
210	0.973644	0.973605	0.973566	0.973546
240	0.973630	0.973590	0.973551	0.973531
270	0.973623	0.973583	0.973544	0.973524
300	0.973618	0.973579	0.973539	0.973519
330	0.973615	0.973576	0.973537	0.973517
360	0.973613	0.973574	0.973534	0.973515
MTBF	351.0875	351.0737	351.0599	351.0529

Table II.
Effect of failure rate of extruder on reliability of the system

$\alpha_2 \rightarrow$ Days $\downarrow$	0.0003	0.0005	0.0007	0.0009
30	0.983150	0.979450	0.975768	0.972104
60	0.980665	0.975639	0.970654	0.965709
90	0.979867	0.974361	0.968909	0.963512
120	0.979568	0.973888	0.968270	0.962713
150	0.979450	0.973708	0.968030	0.962416
180	0.979401	0.973636	0.967937	0.962303
210	0.979379	0.973605	0.967898	0.962258
240	0.979368	0.973590	0.967881	0.962238
270	0.979361	0.973583	0.967873	0.962229
300	0.979357	0.973579	0.967868	0.962224
330	0.979354	0.973576	0.967865	0.962221
360	0.979352	0.973574	0.967863	0.962219
MTBF	352.9831	351.0737	349.1849	347.3155

Effect of failure rate of extruder (α_2) on the reliability of the system. Now, we have studied the effect of failure rate of sub-system, namely, extruder on reliability of the system by varying its failure rate $\alpha_2 = 0.0003$, 0.0005, 0.0007 and 0.0009. The other failure and repair rates have been taken as: $\alpha_1 = 0.0027$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$, $\gamma = 0.0054$, $\beta_1 = 4.800$, $\beta_2 = 0.033$, $\beta_3 = 0.0999$, $\beta_4 = 0.4999$ and $\delta = 12.000$. The reliability of the system is calculated using these data and results are shown in Table II. One can see from this table that reliability and *MTBF* of the system decreases by approximately 1.5 percent with the increase in failure rate from 0.0003 to 0.0009. However, reliability decreases by approximately 0.39 to 1.02 percent with the increase in time from 30 to 360 days.

Effect of failure rate of spray cooling baths (γ) on the reliability of the system. Taking four levels of the failure rate of spray cooling baths, i.e. $\gamma = 0.0050$, 0.0052, 0.0054, 0.0056 and other parameters as: α_1 0.0027, $\alpha_2 = 0.0005$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$,

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$\gamma \rightarrow$ Days ↓	0.0050	0.0052	0.0054	0.0056
30	0.979458	0.979454	0.979450	0.979446
60	0.975651	0.975645	0.975639	0.975632
90	0.974376	0.974368	0.974361	0.974353
120	0.973905	0.973897	0.973888	0.973880
150	0.973725	0.973716	0.973708	0.973699
180	0.973653	0.973645	0.973636	0.973627
210	0.973623	0.973614	0.973605	0.973596
240	0.973608	0.973600	0.973590	0.973582
270	0.973601	0.973592	0.973583	0.973574
300	0.973596	0.973587	0.973579	0.973570
330	0.973593	0.973584	0.973576	0.973567
360	0.973591	0.973582	0.973574	0.973566
MTBF	351.0792	351.0764	351.0737	351.0709

Table III.
Effect of failure rate of spray cooling bath on reliability of the system

$\beta_1 \rightarrow$ Days ↓	4.6	4.8	5.0	5.2
30	0.979390	0.979410	0.979450	0.979490
60	0.975579	0.975599	0.975639	0.975678
90	0.974301	0.974321	0.974361	0.974400
120	0.973829	0.973849	0.973888	0.973927
150	0.973648	0.973668	0.973708	0.973748
180	0.973576	0.973596	0.973636	0.973675
210	0.973546	0.973566	0.973605	0.973644
240	0.973531	0.973551	0.973590	0.973630
270	0.973524	0.973544	0.973583	0.973623
300	0.973519	0.973539	0.973579	0.973618
330	0.973517	0.973537	0.973576	0.973615
360	0.973515	0.973534	0.973574	0.973613
MTBF	351.0655	351.0737	351.0811	351.0881

Table IV.
Effect of repair rate of mixer on reliability of the system

$\beta_1 = 4.800$, $\beta_2 = 0.033$, $\beta_3 = 0.0999$, $\beta_4 = 0.4999$ and $\delta = 12.000$, we have now computed the reliability of the system. This reliability is tabulated in Table III. The table shows that the reliability decreases by approximately 0.6 percent with the increase in time from 30 to 360 days. The reliability decreases from 0.0012 to 0.0026 percent and *MTBF* decreases to 0.0024 percent with the increase in failure rate from 0.0050 to 0.0056.

Effect of repair rate of mixer (β_1) on the reliability of the system. We have now considered the effect of repair rate of the sub-system mixer on the reliability of the system by taking its four levels: $\beta_1 = 4.600$, 4.800, 5.000, and 5.200. The values of other failure and repair rates are taken as: $\alpha_1 = 0.0027$, $\alpha_2 = 0.0005$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$, $\gamma = 0.0054$, $\beta_2 = 0.033$, $\beta_3 = 0.0999$, $\beta_4 = 0.4999$ and $\delta = 12.000$. The reliability of the system is computed by using this data and results are shown in the Table IV. This table shows that by increasing the repair rate from 4.600 to

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Table V.
Effect of repair rate of extruder on reliability of the system

$\beta_2 \rightarrow$ Days $\downarrow$	0.030	0.032	0.034	0.036
30	0.979090	0.979332	0.979566	0.979791
60	0.974817	0.975374	0.975895	0.976384
90	0.973239	0.974003	0.974703	0.975346
120	0.972604	0.973482	0.974275	0.974993
150	0.972342	0.973278	0.974115	0.974866
180	0.972232	0.973195	0.974052	0.974818
210	0.972185	0.973160	0.974025	0.974797
240	0.972163	0.973144	0.974012	0.974786
270	0.972152	0.973135	0.974005	0.974780
300	0.972146	0.973130	0.974001	0.974777
330	0.972142	0.973127	0.973998	0.974774
360	0.972140	0.973125	0.973996	0.974772
MTBF	350.6466	350.9391	351.2011	351.4370

Table VI.
Effect of repair rate of spray cooling bath on reliability of the system

$\delta \rightarrow$ Days $\downarrow$	8.0	10.0	12.0	14.0
30	0.979420	0.979438	0.979450	0.979458
60	0.975588	0.975618	0.975639	0.975653
90	0.974295	0.974334	0.974361	0.974379
120	0.973711	0.973857	0.973888	0.973910
150	0.973623	0.973674	0.973708	0.973732
180	0.973545	0.973599	0.973636	0.973662
210	0.973510	0.973567	0.973605	0.973632
240	0.973493	0.973551	0.973590	0.973619
270	0.973483	0.973543	0.973583	0.973612
300	0.973477	0.973538	0.973579	0.973608
330	0.973473	0.973535	0.973576	0.973605
360	0.973470	0.973532	0.973574	0.973604
MTBF	351.0451	351.0622	351.0737	351.0818

5.200, reliability and *MTBF* increases by approximately 0.007 percent whereas reliability decreases by approximately 0.6 percent with the increase in time from 30 to 360 days.

Effect of repair rate of extruder (β_2) on reliability of the system. Here, Effect of repair rate of extruder on reliability of the system is studied by varying $\beta_2 = 0.30$, 0.032, 0.034 and 0.036. The other parameters are taken as: $\alpha_1 = 0.0027$, $\alpha_2 = 0.0005$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$, $\gamma = 0.0054$, $\beta_1 = 4.800$, $\beta_3 = 0.0999$, $\beta_4 = 0.4999$ and $\delta = 12.000$. The reliability of the system is computed and results are shown in the Table V. We can see from this table that reliability of the system decreases by approximately 0.6 percent with the increase in time 30 to 360 days but reliability increases by approximately 0.07 to 0.27 percent and *MTBF* 0.2 percent with the increase in repair rate of extruder from 0.030 to 0.036.

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$\delta \rightarrow$ $\alpha_2 \downarrow$	8.0	10.0	12.0	14.0
30	0.979420	0.979438	0.979450	0.979458
60	0.975588	0.975618	0.975639	0.975653
90	0.974295	0.974334	0.974361	0.974379
120	0.973711	0.973857	0.973888	0.973910
150	0.973623	0.973674	0.973708	0.973732
180	0.973545	0.973599	0.973636	0.973662
210	0.973510	0.973567	0.973605	0.973632
240	0.973493	0.973551	0.973590	0.973619
270	0.973483	0.973543	0.973583	0.973612
300	0.973477	0.973538	0.973579	0.973608
330	0.973473	0.973535	0.973576	0.973605
360	0.973470	0.973532	0.973574	0.973604
MTBF	351.0451	351.0622	351.0737	351.0818

Table VII. Effect of repair rate of spray cooling bath on reliability of the system using Runge-Kutta Method

Effect of repair rate of spray cooling bath (δ) on reliability of the system. Effect of repair rate of sub-system, spray-cooling bath on reliability of the system has also been studied by us by varying its values as: $\delta = 8.000$, 10.000, 12.000 and 14.000. The failure and repair rates of other sub-systems have been taken as: $\alpha_1 = 0.0027$, $\alpha_5 = 0.0005$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$, $\gamma = 0.0054$, $\beta_1 = 4.800$, $\beta_2 = 0.030$, $\beta_3 = 0.0999$, $\beta_4 = 04999$. The reliability of the system is calculated and results are shown in Table VI. This table reveals that reliability of the system decreases by approximately 0.6 percent with increases in the time from 30 to 360 days. Reliability increases from 0.0039 to 0.014 percent and *MTBF* 0.01 percent with the increase in repair rate of cooling tapes from 8.000 to 14.000.

The results presented in Tables I-VI give the reliability of the plastic-pipe manufacturing plant for various choices of repair and failure rates of the sub-systems. To validate these results, we have also solved the system of differential equations (1) to (5) numerically using Runge-Kutta fourth order method and have computed reliability using equation (13). The results obtained by Runge-Kutta fourth method agree with the results obtained by Matrix calculus method up to six decimal places. This is illustrated in Table VII by calculating the reliability of the system by Runge-Kutta method when the repair rate of the spray cooling bath is varied from $\delta = 8.0$ to $\delta = 14.0$.

4.2 Computations and results for steady state

The effect of various parameters on long run availability is studied in this section. Here, again, we have not considered all the combinations of the parameters governing the system but some of them, which have greater impact on the system, have been studied.

Effect of failure rates of mixer and extruder on long run availability of system. Taking, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$, $\gamma = 0.0054$, $\beta_1 = 4.800$, $\beta_2 = 0.033$, $\beta_3 = 0.0999$, $\beta_4 = 0.4999$ and $\delta = 12.000$ and varying the failure rates of mixer and extruder as: $\alpha_1 = 0.002$, 0.003, 0.004, 0.005; $\alpha_2 = 0.0004$, 0.0005, 0.0006 and 0.0007, we have computed the long run availability of the system using (20). The long run availability

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of the system for the various combinations of failure rates of mixer and extruder is given in Table VIII. A critical study of table reveals that the failure rate (α_2) of extruder affects the long run availability by approximately 1.7 percent whereas failure rate (α_1) of raw mixer affects by approximately 0.010 percent.

Effect of repair rates of mixer and extruder machine on long run availability of system. Taking, $\alpha_1 = 0.0027$, $\alpha_2 = 0.0005$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$, $\gamma = 0.0054$, $\beta_3 = 0.0999$, $\beta_4 = 0.4999$ and $\delta = 12.000$ and using (20), we have computed the long run availability of the system for combinations of repair rates of mixer and extruder as: $\beta_1 = 4.6,\ 4.8,\ 5.0,\ 5.2$; $\beta_2 = 0.03,\ 0.032,\ 0.034,\ 0.036$. The computed long run availabilities are given in Table IX. A critical study of table reveals that repair rate (β_2) of extruder affects the long run availability of the system by approximately 0.27 percent and repair rate of raw mixer (β_1) affects it by approximately 0.0067 percent. As such, effect of β_2 is more in comparison with β_1.

Effect of failure and repair rates of cooling taps on long run availability of system. Taking, $\alpha_1 = 0.0027$, $\alpha_2 = 0.0005$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$, $\beta_1 = 4.800$, $\beta_2 = 0.033$, $\beta_3 = 0.0999$ and $\beta_4 = 0.4999$, we have computed the long run availability of the system for the combinations of failure and repair rates of cooling taps as: $\delta = 8.0$, 10.0, 12.0, 14.0, $\gamma = 0.0050$, 0.0052, 0.0054, 0.0056. The results are given in Table X. A close study of this table reveals that failure rate of cooling taps (γ) affects long run

Table VIII.
Effect of failure rate of sub-systems mixer and extruder on long run availability of the system

$\alpha_1 \rightarrow$ / $\alpha_2 \downarrow$	0.0025	0.0027	0.0029	0.0030
0.0003	0.979388	0.979348	0.979308	0.979288
0.0005	0.973608	0.973569	0.973530	0.973510
0.0007	0.967897	0.967858	0.967819	0.967800
0.0009	0.962253	0.962214	0.962175	0.962156

Table IX.
Effect of repair rate of sub-systems mixer and extruder on long run availability of the system

$\beta_1 \rightarrow$ / $\beta_2 \downarrow$	4.6	4.8	5.0	5.2
0.030	0.972112	0.972135	0.972156	0.972176
0.032	0.973097	0.973120	0.973142	0.973161
0.034	0.973968	0.973992	0.974013	0.974033
0.036	0.974744	0.974767	0.974789	0.974808

Table X.
Effect of failure and repair rates of sub-systems spray cooling bath on long run availability of the system

$\delta \rightarrow$ / $\gamma \downarrow$	8.0	10.0	12.0	14.0
0.0050	0.973486	0.973545	0.973585	0.973613
0.0052	0.973474	0.973536	0.973577	0.973606
0.0054	0.973462	0.973526	0.973569	0.973600
0.0056	0.973451	0.973517	0.973561	0.973593

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availability by approximately 0.0036 to 0.002 percent but its repair rate (δ) affects it by approximately 0.015 percent.

Effect of failure and repair rates of extruder on long run availability of system. Now, taking, $\alpha_1 = 0.0027$, $\alpha_3 = 0.0009$, $\alpha_4 = 0.0011$, $\gamma = 0.0054$, $\beta_1 = 4.8$, $\beta_3 = 0.0999$, $\beta_4 = 0.4999$ and $\delta = 12.0$, we have computed the long run availability of the system for the combination of failure and repair rates of extruder as: $\alpha_1 = 0.0003, 0.0005, 0.0007, 0.0009$; $\beta_2 = 0.030, 0.032, 0.034, 0.036$. The results of this computation given in Table XI depict the effect of failure and repair rates of extruder on the long run availability of the system. We can conclude from this table that repair rate of extruder (β_2) affects the long run availability more than the failure rate of extruder (α_2). The affect of α_2 on availability varies from 0.16 to 0.48 percent whereas that of β_2 varies from 1.6 to 2.0 percent.

Analysis of results

A comparative study of Tables I-VII and Tables VIII-XI reveals that sub-system *B*, i.e. extruder has the maximum impact on the reliability and long run availability of whole of the system. The effect of failure and repair rates of sub-system *B* on the reliability of system has also been presented graphically in Figures 3-4. The other sub-systems are almost equal contributors towards the reliability and long run availability of the system. This trend has been observed for all the data collected from the plant. Therefore, it is recommended that management should pay more attention to sub-system *B* so that the overall reliability and long run availability of the system may improve.

$\alpha_1 \rightarrow$ $\beta_2 \downarrow$	0.0003	0.0005	0.0007	0.0009
0.030	0.978476	0.972135	0.965875	0.959696
0.032	0.979075	0.973120	0.967238	0.961426
0.034	0.979604	0.973992	0.968443	0.962957
0.036	0.980075	0.974767	0.969517	0.964323

Table XI. Effect of failure and repair rates of sub-system extruder on long run availability of the system

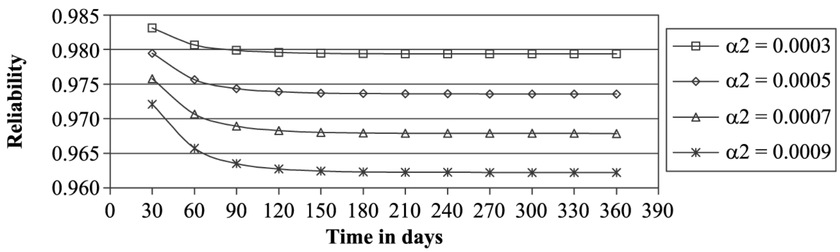

Figure 3. Effect of failure rate of extruder

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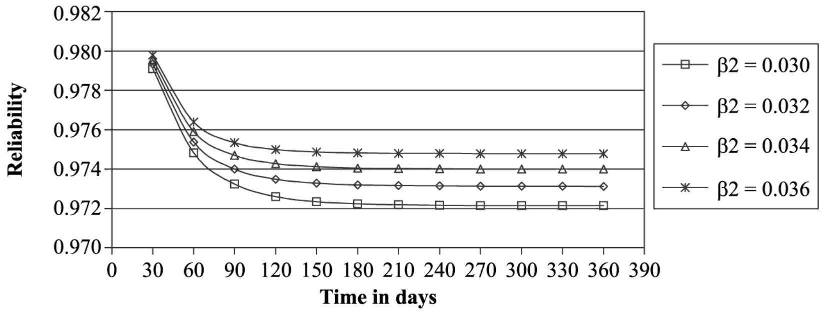

Figure 4.
Effect of repair rate of extruder

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